PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.
INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two:

Calculator-assumed

Your Name _____

Your Teacher's Name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Marks	Max
8		6	15		9
9		9	16		7
10		9	17		7
11		8	18		8
12		3	19		9
13		9			
14		10	Total		(94)

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	47	33
Section Two: Calculator-assumed	12	12	100	94	67
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **67% (94 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(2,4 = 6 marks)**

Four points have coordinates $A(-11, 7), B(4, -3), C(7, 9)$ and $D(s, t)$.

(a) If B is the midpoint of A and D, determine the coordinates of point D. (2 marks)

(b) Determine the equation of the line that is perpendicular to AB and that passes through C in the form $ax + by + c = 0$, where a, b and c are integers and $a > 0$. (4 marks)

Question 9 **(1,1,1,1,1,2,2 = 9 marks)**

The Amy car insurance company classifies its drivers according to age and gender, as shown in the following table.

		GENDER		
		Male	Female	Total
AGE	Under 25	0.15	0.12	0.27
	25 or Over	0.45	0.28	0.73
	Total	0.6	0.4	1

(a) Determine the probability that a randomly chosen driver is:

(i) Female. (1 mark)

(ii) Female and Under 25. (1 mark)

(iii) Male, or 25 or Over. (1 mark)

(iv) Female, given the driver is Under 25. (1 mark)

(v) 25 or Over, given the driver is Male. (1 mark)

For the different classes of drivers above, the probability *p,* of having at least one accident in a year, is given in the table below.

	Male	Female
Under 25	0.09	0.06
25 or Over	0.04	0.02

(b) (i) Determine the probability that a randomly chosen driver has at least one accident in a year. (2 marks)

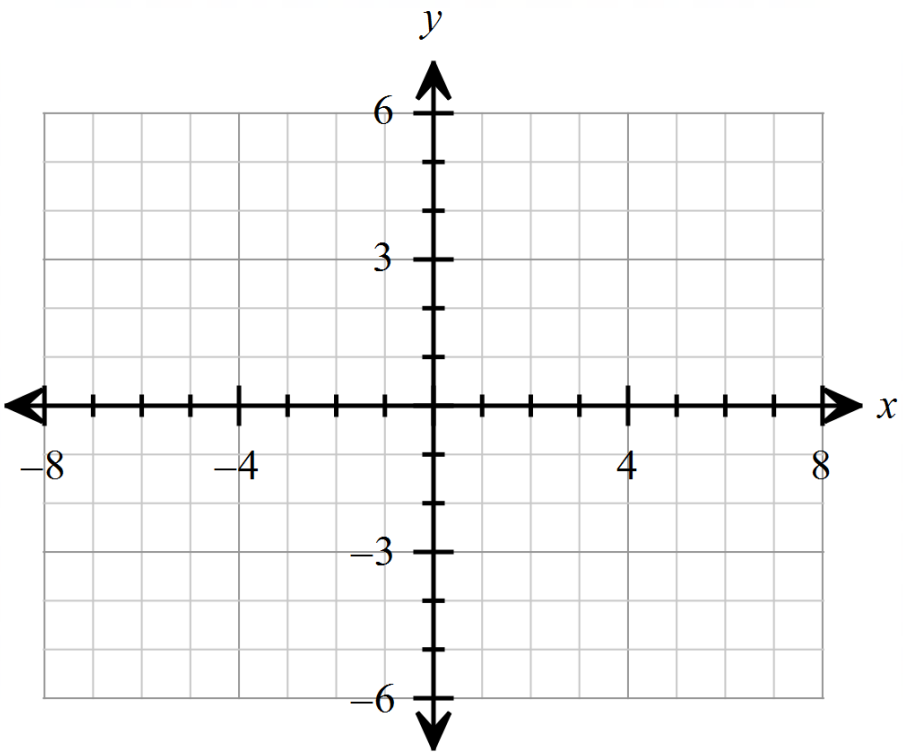
(ii) If a driver has at least one accident in a year, what is the probability that the driver is Male and Under 25? (2 marks)

Question 10 **(3,4,2 = 9 marks)**

A function defined by $f(x) = \frac{a}{x+b}$, where a and b are constants, passes through the points $(-8, 0.2)$ and $(2, -0.6)$.

(a) Determine the value of a and the value of b. (3 marks)

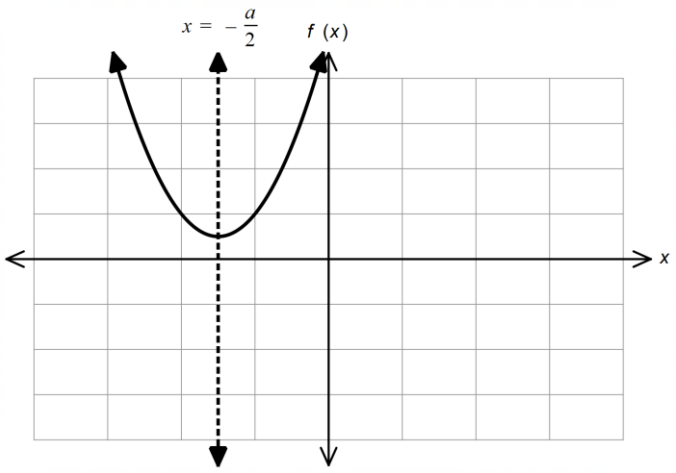
(b) Draw the graph of $y = f(x)$ on the axes below, clearly indicating the coordinates of all axes intercepts and equations of any asymptotes. (4 marks)



(c) State the equations of all asymptotes of the graph of $y = f(2x) - 3$. (2 marks)

Question 11 **(3,2,3 = 8 marks)**

The function $f(x) = x^2 + ax + 5$ is graphed below with a line of symmetry at $x = -\frac{a}{2}$.



(a) The turning point of $f(x)$ exists in the second quadrant only.
Determine all the possible values of a. (3 marks)

(b) Sketch the function $y = g(x)$ such that $g(x) = -f(x)$ on the same axes above. (2 marks)

(c) If $a = 8$ change $f(x)$ into the form $(x+b)^2 + c$ and state the turning point and line of symmetry.

(3 marks)

Question 12 **(3 marks)**

A circle has equation $x^2 + y^2 + 4x - 6y = 36$

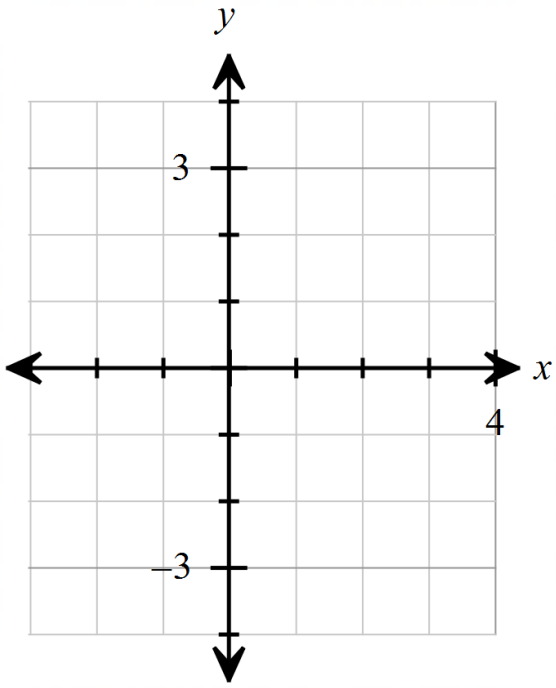
Determine the centre and radius of the circle. (3 marks)

Note: 0 mark for "Centre is (-4, 6) and radius is 6".

Question 13 **(3,3,3 = 9 marks)**

The graph of the cubic polynomial $y = f(x)$ passes through the points $(3,0)$, $(0,-3)$ and has a local maximum at $(1,0)$ and can be written in the form $f(x) = (x+e)^2(x+g)$ where e & g are constants.

(a) Use the above information to sketch the graph of $y = f(x)$ on the axes below. (3 marks)



Let $f(x) = x^3 + bx^2 + cx + d$, where b, c and d are constants.

(b) Determine the value of each of the constants b, c and d. (3 marks)

(c) Another cubic polynomial is defined by $g(x) = x^3 - 8x^2 + ax - 6$. Determine the value(s) of the constant a so that the graphs of $y = f(x)$ and $y = g(x)$ do not intersect. (3 marks)

Question 14 **(4,1,1,2,2 = 10 marks)**

A class of 30 students are surveyed on which ATAR science subject they chose.

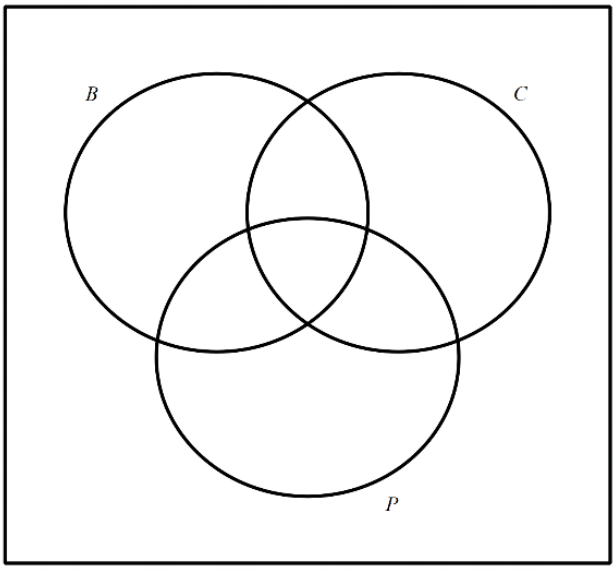
B = Biology ***C = Chemistry*** ***P = Physics***

Given the information:

$n(B) = 16$ $n(C) = 18$ $n(P) = 13$

$n(B \cap C) = 10$ $n(B \cap P) = 3$ $n(C \cap P \cap B') = 6$ $n(B \cap C \cap P) = 2$

(a) Use the given information to complete all regions of the Venn diagram below. (4 marks)



(b) Show using set notation the set of students who chose Physics only. (1 mark)

(c) Show using set notation the set of students who did not choose any science subjects. (1 mark)

(d) A student is chosen at random from the class. Calculate:

(i) $P(C|B)$ (2 marks)

(ii) $P(\overline{B \cup P})$ (2 marks)

Question 15 **(1,2,1,2,3 = 9 marks)**

A committee of 5 is chosen to plan the ball for 2023.
If there are 10 year 11's and 5 Year 12's to choose from, determine the number of ways of selecting the committee given the following restrictions.

(a) If there are no restrictions. (1 mark)

(b) It must contain 3 Year 11's and 2 Year 12's. (2 marks)

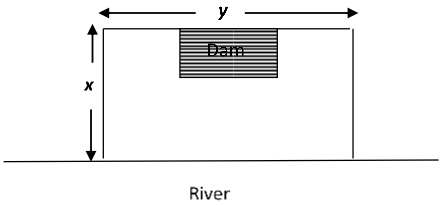
(c) It must contain all Year 11's. (1 mark)

(d) It must contain at least 3 Year 11's. (2 marks)

(e) It must contain at least 1 from each Year. (3 marks)

Question 16 **(1,2,2,2 = 7 marks)**

A farmer uses 1250 metres of fencing to construct a rectangular shaped field. His land is located alongside a river, so he only needs to fence three sides of the field. There is also a rectangular shaped dam of area 5000 m^2 within the field.



Let *x* be the width of the two shorter sides of the field.

(a) If *y* is the length of the field, express *y* in terms of *x*. Let *A* m^2 be the area of the farmable land inside the fence. (1 mark)

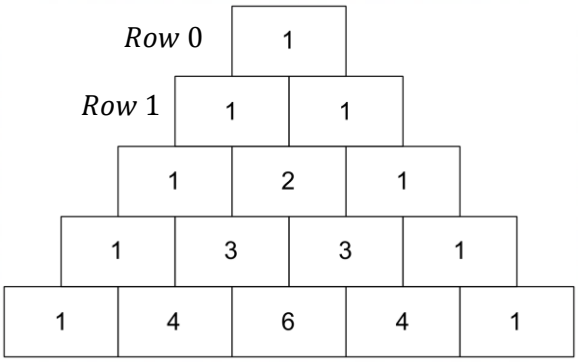
(b) Show that $A = -2x^2 + 1250x - 5000$ (2 marks)

(c) Use your calculator to find the coordinates of the turning point of the graph. Give your answer to 1 decimal place. (2 marks)

(d) Find the dimensions of the field which gives the maximum farmable area, and state the maximum area? Give your answer to 1 decimal place. (2 marks)

Question 17 **(2,2,1,2 = 7 marks)**

(a) Pascal's triangle is shown below. State the next two rows of the triangle. (2 marks)



(b) Solve for x using Pascal's triangle. (2 marks)

(i) ${}^{4}\mathbf{C}_{x} = 6$

(ii) ${}^{6}\mathbf{C}_{x+1} = {}^{6}\mathbf{C}_{x-3}$

(c) Consider the binomial expansion of $(2a - b)^{7}$.

(i) Which row and which term in Pascal's triangle would you use to determine the coefficient that goes with the term a^4b^3 ?

(1 mark)

(ii) Show that the coefficient of a^4b^3 is -560. (2 marks)

Question 18 **(4,4, = 8 marks)**

(a) The graph of the quadratic function $f(x) = a(x+b)^2 + c$ has roots at $x = -1$ and $x = 9$ and the range of $f(x)$ is $y \geq -50$. Use an algebraic method to determine $f(0)$. (4 marks)

(b) The area of square B is 303.5 cm^2 greater than twice the area of square A, (i.e $B^2 = 2A^2 + 303.5$ cm^2) and the difference in the perimeters of the two squares is 50 cm, Determine the least possible area of square A, the smaller of the squares and **show all steps** of your working.

(4 marks)

Question 19 **(1,4,4 = 9 marks)**

A chemical manufacturer has **15%** and **40%** acid solutions (i.e., 15% and 40% by volume is acid respectively) available in stock. The manufacturer needs to make up solutions from a mixture of 15% and 40% solutions. Let x be the amount of 15% solution required. Let y be the amount 40% solution required.

The manufacturer has an order for 500 litres of a 25% acid solution.

(a) How much acid is required to produce 500 litres of 25% acid solution? (1 mark)

(b) Determine the amount of each solution required. (4 marks)

(c) Determine x and y if the manufacturer has an order for p litres of a q% acid solution. (4 marks)

End of Questions

Supplementary page

Question number: ____